UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF ELECTRICAL ENGINEERING

EE.351: Spectrum Analysis and Discrete-Time Systems

FINAL EXAM, 9:00AM–12:00PM, December 13, 2004 **(closed book)**

Examiner: Ha H. Nguyen

There are five questions. All questions are of equal value but not necessarily of equal difficulty. Full marks shall only be given to solutions that are properly explained and justified.

The last page contains useful information and formulas. Note, however, that you may not need all of them.

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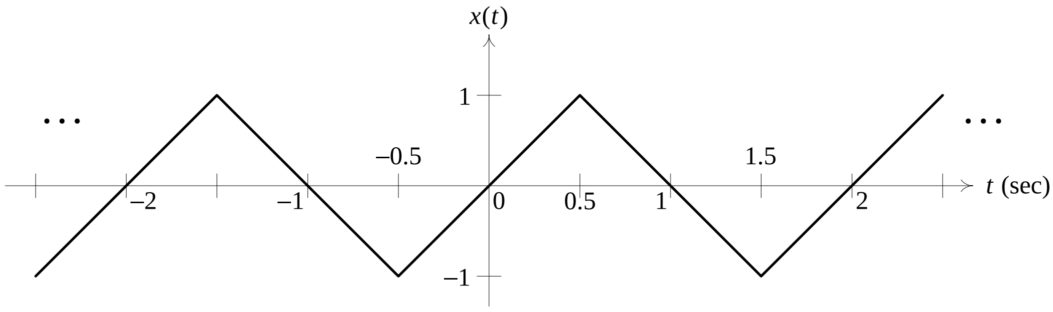
*Name:*______________________________ *Student Number:*________________

Please do not turn this cover page until you are told to do so.

MARKER'S USE ONLY

Question 1:________	$/12$
Question 2:________	$/12$
Question 3:________	$/12$
Question 4:________	$/12$
Question 5:________	$/12$
________ TOTAL	$/60$

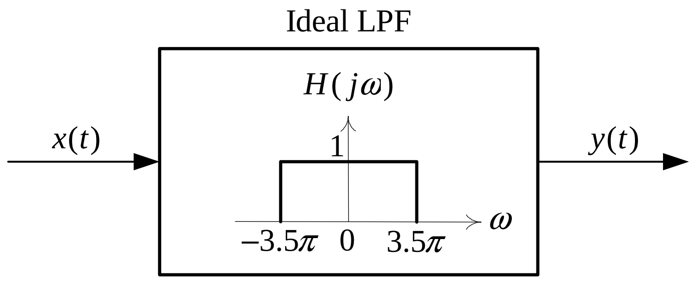
1. *(FS of a continuous-time signal)* Consider the following periodic continuous-time signal:



[2] (a) What are the fundamental period and fundamental frequency of $x(t)$?

[5] (b) Compute the trigonometric Fourier series coefficients for the signal $x(t)$.

[5] (c) The signal $x(t)$ is passed through an ideal lowpass filter with a cutoff frequency of $\omega_c = 3.5\pi$ as shown below:



Neatly draw the magnitude and phase spectra of the output signal $y(t)$. Also find the average power of $y(t)$.

Hint: The response of an LTI system to a periodic CT signal $x(t)$ is given by:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \longrightarrow y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}$$

2. *(FS of a discrete-time signal)* Consider the following discrete-time periodic signal:

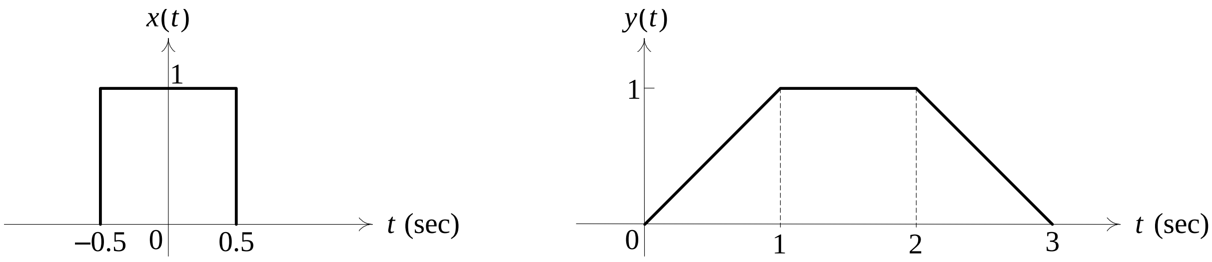
$$x[n] = 1 + \sin\left(\frac{2\pi}{5}n\right) + 3\cos\left(\frac{2\pi}{5}n\right) + \cos\left(\frac{4\pi}{5}n + \frac{\pi}{2}\right)$$

[2] (a) What are the fundamental frequency and fundamental period of $x[n]$?

[6] (b) Find the exponential Fourier series coefficients for $x[n]$.

[4] (c) Plot the magnitude and phase spectra of $x[n]$ over any two periods.

3. *(FT of continuous-time signals)* Consider two signals $x(t)$ and $y(t)$ shown below.

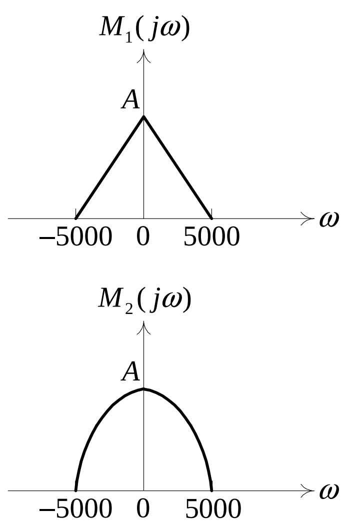


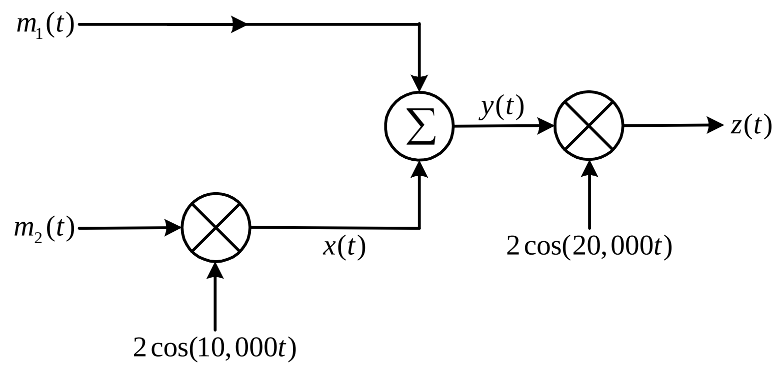
[3] (a) Show that the Fourier transform of the signal $x(t)$ is $X(j\omega) = \dfrac{2\sin(\omega/2)}{\omega}$.

[6] (b) Using the Fourier transform of $x(t)$, the time-shifting and differentiation properties of the Fourier transform, find the Fourier transform of $y(t)$.

[3] (c) It is known that the signals $x(t)$ and $y(t)$ are the input and output of an LTI system, respectively. What is the transfer function $H(j\omega)$ of this LTI system?

4. *(Frequency-Shifting Property of FT)* The manager of your division asked you to design a communication system to transmit two signals $m_1(t)$ and $m_2(t)$ simultaneously over the same channel. It is also required that the design makes use of the company's available carrier generators (i.e., oscillators) $\cos(10,000t)$ and $\cos(20,000t)$. After examining the spectra of the two signals, you present the following block diagram for the transmitter:



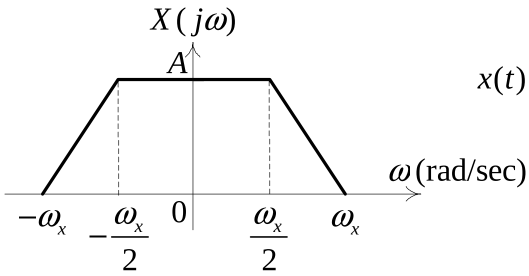


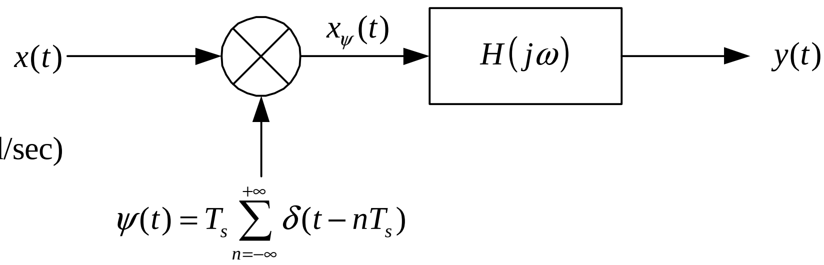
[5] (a) Plot the spectra of three signals $x(t)$, $y(t)$ and $z(t)$ identified in the above block diagram. Clearly identify all relevant frequencies.

[1] (b) What is the minimum bandwidth of the channel you need?

[6] (b) To convince your manager that your design is valid, show and explain to him a block diagram of a receiver that can perfectly recover the signals $m_1(t)$ and $m_2(t)$ from the modulated signal $z(t)$. The receiver must also use the two available oscillators $\cos(10,000t)$ and $\cos(20,000t)$.

5. *(Sampling)* A block diagram of impulse sampling is shown below.

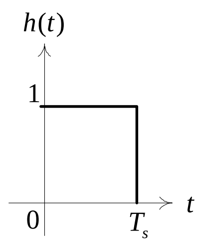




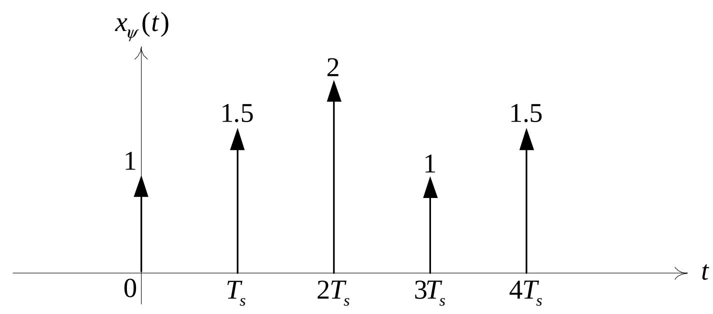
[4] (a) What is the minimum sampling frequency to prevent aliasing? Justify your answer by plotting the spectrum of $x_\psi(t)$ and explaining how $x(t)$ can be recovered from $x_\psi(t)$.

[5] (b) Draw the spectrum of $x_\psi(t)$ when the sampling frequency is $\omega_s = \omega_x$. Also draw the spectrum of $y(t)$ if $H(j\omega)$ is an ideal lowpass filter with a cutoff frequency $\omega_c = \omega_x$.

[3] (c) Instead of the ideal lowpass filter, consider the filter whose impulse response shown in Figure (a) as a reconstruction filter. Find and plot the reconstructed signal $y(t)$ if $x_\psi(t)$ is as shown in Figure (b) over the interval $[0, 4T_s]$.



(a)



(b)

Potentially Useful Facts:

- FS representation of CT periodic signals (CTFS):

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega t} = a_0 + 2\sum_{k=1}^{\infty}[B_k \cos(k\omega t) - C_k \sin(k\omega t)]$$

$$\begin{aligned} a_k &= B_k + jC_k, \quad |a_k| = \sqrt{B_k^2 + C_k^2}, \quad \angle a_k = \tan^{-1}(C_k/B_k) \\ a_k &= \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega t}dt \\ B_k &= \frac{1}{T_0}\int_{T_0} x(t)\cos(k\omega t)dt, \quad C_k = -\frac{1}{T_0}\int_{T_0} x(t)\sin(k\omega t)dt \end{aligned}$$

  Remark: The trigonometric form only applies for real-valued signals.

- FS representation of DT periodic signals (DTFS):

$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n}, \quad a_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\omega_0 n}$$

- The *complex-conjugate symmetry* of the Fourier series coefficients of real-valued periodic signals:

$$\boxed{a_{-k}^* = a_k, \qquad a_{-k} = a_k^*}$$

- *Parseval's relation*:

$$\boxed{\frac{1}{T_0}\int_{T_0} |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2}$$

- Identities:

$$\begin{aligned} e^{j\omega t} &= \cos(\omega t) + j\sin(\omega t) \\ e^{-j\omega t} &= \cos(\omega t) - j\sin(\omega t) \end{aligned}$$

- Indefinite integrals:

$$\int \cos(at)dt = \frac{1}{a}\sin(at), \quad \int \sin(at)dt = -\frac{1}{a}\cos(at)$$

$$\int t\cos(at)dt = \frac{1}{a^2}[\cos(at) + at\sin(at)], \quad \int t\sin(at)dt = \frac{1}{a^2}[\sin(at) - at\cos(at)]$$

- Fourier transform of CT signals:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}d\omega, \quad X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$$

- Fourier transform pairs/properties:

$$\begin{aligned} \cos(\omega_0 t) &\stackrel{\mathcal{FT}}{\longleftrightarrow} \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0) \\ \delta(t) &\stackrel{\mathcal{FT}}{\longleftrightarrow} 1 \\ x(t - t_0) &\stackrel{\mathcal{FT}}{\longleftrightarrow} e^{-j\omega t_0}X(j\omega) \\ \frac{d}{dt}x(t) &\stackrel{\mathcal{FT}}{\longleftrightarrow} j\omega X(j\omega) \\ \cos(\omega_0 t)x(t) &\stackrel{\mathcal{FT}}{\longleftrightarrow} \frac{1}{2}X(j(\omega - \omega_0)) + \frac{1}{2}X(j(\omega + \omega_0)) \\ x(t)\psi(t) &\stackrel{\mathcal{FT}}{\longleftrightarrow} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s)) \quad \text{(Impulse sampling)} \end{aligned}$$